9.1 Simple Harmonic Motion

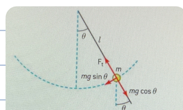
- **Angular speed or frequency (ω)**
    - The angular speed, ω, is the rate of change of angle with time.
        - Also called, angular frequency.
        - Measured in radians per second ($rad\,s^{-1}$).
- **Circular motion an SHM**
    - For constant angular speed $\theta = \omega t$.
- **The relationship between displacement, velocity, and acceleration**
    - In circular motion, the displacement takes the form of: $x = x_0 \sin \omega t$ or $x = x_0 \cos \omega t$ depending on the timing.
        - x_0 is the amplitude, and ω is the rotational frequency.
    - In SHM, the acceleration will be given by the equation: $a = -\omega^2 x$
        - Due to the fact that acceleration will be in the opposite direction of the displacement.
    - The maximum velocity is given by: $v_0 = x_0 \omega$.
    - Max acceleration is given by: $a_0 = x_0 \omega^2$.
- **The SHM equation and ω^2**
    - The acceleration is defined (for SHM) as motion in which the acceleration is proportional to the displacement from a fixed point and is always directed towards that fixed point.
    - Therefore, $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$ as $f = \frac{1}{T}$.
- **The velocity equation**
    - $v = \pm \omega \sqrt{x_0^2 - x^2}$
- **Simple Harmonic systems**
    - **The simple pendulum**
        - 

Figure 4 Restoring force for simple pendulum.

        - A condition of a system oscillating simple harmonically is that there is a restoring force that is proportional to the displacement from the equilibrium position.
            - meaning that $F = -m\omega^2 x$.
        - The mass on the string is in equilibrium when the tension from the string is equal to the weight.
        - The restoring force is given by: $a = -\left(\frac{g}{l}\right)x$ or $\omega = \left(\frac{g}{l}\right)$
    - **Mass - spring system**
        - The mass will exchange elastic potential energy (at full extension and compression) with kinetic energy (as it passes through the equilibrium position).
        - When a spring (constant "k") is extended by x from its equilibrium point there will be a restoring force acting on the mass.
            - This is given by: $F = -kx$.
            - The force will be in the opposite direction to the extension.
        - The period of a mass on a spring is given by: $T = 2\pi\sqrt{\frac{m}{k}}$.
    - **Energy in SHM systems**
        - The maximum energy of the system is the kinetic energy when the gravitational potential energy is zero.
            - Given by: $E_T = \frac{1}{2} m \omega^2 x_0^2$.
        - The gravitational potential energy is given by: $E_p = \frac{1}{2} m \omega^2 x^2$.
        - The velocity is given by: $v = x_0 \omega \cos \omega t$
        - Kinetic energy (in terms of time) is given by: $E_k = \frac{1}{2} m x_0^2 \omega^2 \cos^2 \omega t$
        - Potential energy (in terms of time) is given by: $E_p = \frac{1}{2} m x_0^2 \omega^2 \sin^2 \omega t$.

9.2 Single slit diffraction

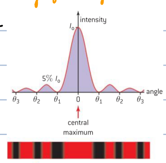
- **Graph of intensity against angle**
    - 

Figure 1 Variation of intensity with angle for a diffraction pattern.

    - $\theta_1, \theta_2, \theta_3$ are the angles with the straight-through position made by the minima.

- The single-slit equation

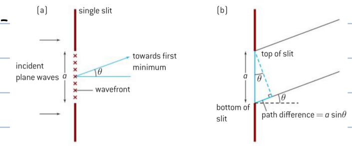


Figure 2 Deriving the single slit equation.

- Figure 2a shows a single slit of width a.
  - Each "x" will behave as a wave source.
- For the two waves coming from the edges of the slit they will have a path difference of $a \sin\theta$.
  - Waves coming from halfway in the slit will have a path difference of $\frac{a}{2}\sin\theta$ with waves from each edge.
  - When the waves will have a path difference of half a wavelength, then waves from halfway from the edge of a slit will undergo destructive interference with waves from the bottom.
- Therefore, whenever there is a path difference of $\frac{a}{2}\sin\theta = \frac{\lambda}{2}$, there is destructive interference between a wave from the top of the slit with one from the bottom.
- Whenever small angles are used $\sin\theta$ can be approximated to θ arriving at: $\theta = \frac{\lambda}{a}$.
  - The position of other minima will be given by $\theta = \frac{n\lambda}{a}$ (where $n = 2, 3, 4 \ldots$).
- Worked example
  - At a single slit there won't just be one light source, but instead there will be multiple smaller sources that will cancel one another out as light from the top of the slit will have a path difference of $\frac{\lambda}{2}$ with light from the bottom of the slit causing destructive interference.
  - $\lambda = 630 \cdot 10^{-9}$ m

    $\underline{7.35 \cdot 10^{-4}}$ m

    $\frac{\lambda}{d} = \theta \quad \theta = \frac{\Lambda}{D}$

    $\frac{\lambda}{d} = \frac{\Lambda}{D}$

    $d = \frac{\lambda D}{\Lambda}$

- Single splits with monochromatic and white light
  - Monochromatic light means that its a single color.
  - There will be a difference between the central maximum and the angular separation of successive secondary maxima depend on the wavelength of the light.
    - This is why there are colors in the fringes after the central one for white light.
    - Blue is the less deviated color as it has the shortest wavelength.

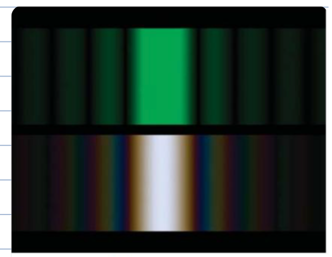


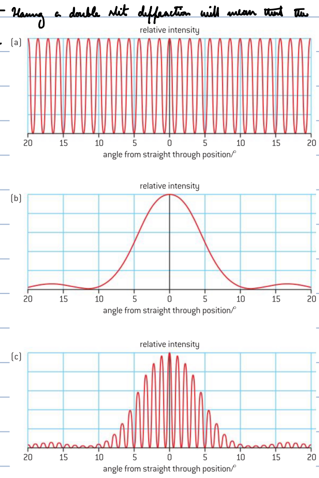
Figure 3 Single slit with monochromatic and white light.

9.3 Interference

- Intensity variation with the double slit
  - As seen in the image below the dark and bright fringes are equally spaced.
  -

Figure 1 Double-slit diffraction pattern for light from He–Ne laser.

- Having a double slit diffraction will mean that the intensity of the interference pattern is not constant, but is modified by the diffraction pattern to produce the intensity.
-



- Figure a shows what the relative intensity would vary for a double-slit interference pattern without diffraction.
- Figure b shows the relative intensity with angle for a single slit.
- Figure c shows the superposition of the two effects so that the single-slit diffraction pattern behaves as the envelope of the interference pattern.

- $s = \frac{\lambda D}{d}$ where D is the distance from the double slit to the screen and d is the separation of the slits.
    - The value of s is used with the diffraction graph.

- **Multiple-slit interference**
    - Using more than two slit will mean that the bright fringes remain in their current position, but become sharper.
        - meaning that they're narrower and their intensity is proportional to the square of the number of slits.
    - The reason that the middle fringe is so bright is because of the fact that all the light from the slits will be in phase when they reach it.
        - the further away you are from the central maximum the more likely the light is out of phase.
    - Increasing the number of slits to 100 there will be a slit with a path difference of $\frac{\lambda}{2}$ from the first.
        - When the light will interact near the middle will result in destructive interference.
        - This will cause the central maxima to become more and more narrow as more slits are added.
            - It will also decrease the width of other maxima.
        - Increasing the number of slits will also increase the intensity of all maxima as more light is incident at the maxima.
    -

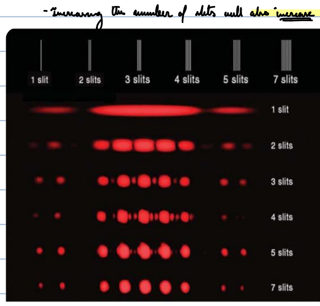


Figure 3 The effect of increasing the number of slits on an interference pattern

- The single slit diffraction envelopes all other diffraction patterns.
    - Meaning that they will fit in the single slit pattern.
-

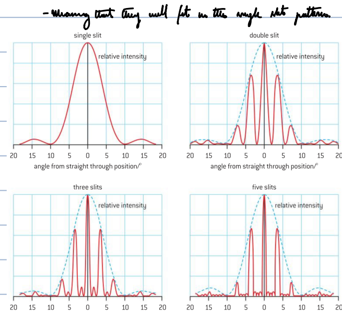


Figure 4 The effect of increasing the number of slits on variation of intensity.

- **The diffraction grating**
    - A diffraction grating is an instrument which is used to produce slits to observe the interference pattern.
        - They're used to produce optical spectra.
    - A grating will contain a large amount of slits, usually 600 lines per millimeter.
    - When light is incident on a grating it produces interference maxima at angles θ, and is given by:
        - $n\lambda = d \sin\theta$
    - The split spacing is small, which makes the angle θ large for a fixed wavelength of light and n.
        - This means $\sin\theta$ has to be used since the angle will be too large to approximate.
    - The waves will have a maximum as the slits are so narrow and far apart from the screen that the angles are almost identical.
        - Furthermore, the waves will be in phase and give a maximum.
            - They're in phase because the path difference is negligible.
    - For a diffraction grating the equation will be: $n\lambda = d \sin\theta$
        - "n" is the order of the maximum, where the central one is zero, and 1 for the first maxima on each side of the center.
            - d is the grating spacing.
    - The further you are away from the maximum the wider and less intense the maximums become.
- **Grating spacing and number of lines per mm**
    - $d = \frac{1}{n}$, where n is the number of lines per mm, and d is the grating spacing.

- n must first be in number of lines per millimeter by multiplying by 1000 before taking the reciprocal.

- **Worked example**

- $n = 600$ $\quad n\lambda = (\sin\theta)d$

$d = \frac{1}{n}$ $\quad \lambda = \frac{d\sin\theta}{n}$

$= \frac{1}{600{,}000}$ $\quad = \frac{(1.67\cdot10^{-6})(\sin 42.5°)}{2}$

$d = 1.67\cdot10^{-6}$ m $\quad = \underline{5.64\cdot10^{-7}\text{ m}}$

- **Interference by division of amplitude**
  - Division of amplitude is a method of achieving interference using two waves that have come from the same point on a wavefront.
    - Each wave will have a portion of the amplitude of the original wave.
  - For division of amplitude to occur, the source of the wave must be much larger than the slit used for the division of wavefront interference.
    - The image will be "localized".

- **Thin film interference** *

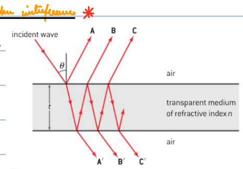


Figure 9 Interference at parallel-sided thin film.

- The wave incident at an angle θ to the normal to the surface of a film of transparent material (of low density) having refractive index n.
- θ and t are very small.
- The incident wave partially reflects at the top of the surface, and partially refracts into the film.
  - Once the refracted wave reaches the other part of the film, it will again partially reflect, and partially refract into the air below the film.
- For the light reflected by the film when $2tn = m\lambda$ there is destructive interference, and $2tn = (m + \frac{1}{2})\lambda$ there's constructive interference.
- **Worked example**
  - Thin film interference.
  - $n = 1.45$, $n = 1.33$ $\lambda = 650\cdot10^{-9}$ m
    - Since the oil will be denser than the water, as light moves from water into oil, it will further bend from the normal causing for the light to separate more into its individual colors.

- **Thin film interference**

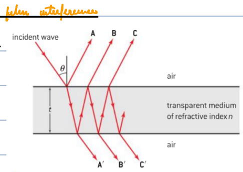


Figure 9 Interference at parallel-sided thin film.

- The light is incident on transparent material, with refractive index n, (e.g. low density oil or detergent) at an angle θ to the normal.
  - θ and t (the size of the thin film) are both very small so that the incident wave is effectively normal to the surface.
- The incident wave is partially reflected and refracted.
  - The refracted wave will then partially reflect when it reaches the other end of the film, it will also partially refract into the air below the film.
- **Waves reflected by the film**
  - The case of line A, it has been reflected.
    - The reflection is from an optically denser medium and therefore will undergo a phase change of π radian ($\frac{\lambda}{2}$).
  - The light wave B will travel an optical distance of $2tn$ (where n is the refractive index).
    - Therefore the optical distance between A and B is $\underline{\underline{2tn}}$.
  - If there is no phase change then this optical distance would equal $m\lambda$ for constructive interference.
    - Since there is a phase change when A was reflected, then there will be destructive interference between A & B.
  - When light is reflected from the film when: $2tn = m\lambda$ there will be destructive interference, and when $2tn = (m + \frac{1}{2})\lambda$ there will be constructive interference.
- **Iridescent thin film interference**
  - When light hits a boundary with a higher refractive index it will undergo a phase change of π radians as it reflects off the boundary.
    - The light that is reflected will have a path difference of $\frac{\lambda}{2}$ compared to the original wave.
    - When light reflects from a surface there is a phase change of π radians, only if the surface causing the reflection is more optically dense.
  - Some of the light initially incident on the thin film will be refracted.

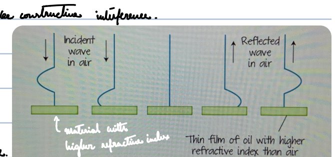
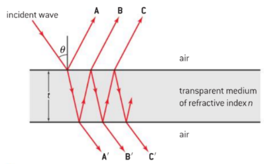


Figure 1. When light reflects from a boundary where the refractive index increases, the phase changes by π radians, so the wave inverts.

- When it hits the bottom of the film it will be partially reflected and refracted into air.
- Since air will have a lower refractive index the light reflected back into the film won't undergo a phase change.
-



▲ Figure 9 Interference at parallel-sided thin film.

- The waves A and B will interfere destructively if they're out of phase.
- The waves in the film travel a distance of $2t$.
- Light in the film will travel more slowly by the refractive index of the oil.
- Combining the speed and distance will result in the optical distance travelled by a wave in a thin film as: $2dn$.
- But since waves A and B are $\frac{\lambda}{2}$ out of phase, the formula: $2dn = m\lambda$ will give the destructive interference pattern.
  - Constructive interference will occur when $2dn = (m + \frac{1}{2})\lambda$.

- Worked example
  - $\lambda = 600 \cdot 10^{-9}$ m $d = 3.1 \cdot 10^{-6}$ m, $n = 1.5$

$2dn = m\lambda$

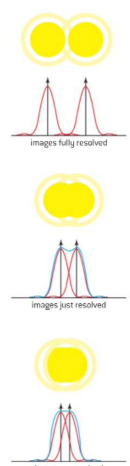
$m = \frac{2(3.1 \cdot 10^{-6})(1.5)}{(600 \cdot 10^{-9})}$

$m = 15.5$ not whole number = constructive interference.

constructive interference.

9.4 Resolution

- Resolution is defined as: Resolution is the ability of an imaging system to be able to produce two separate distinguishable images of two separate objects.
- Diffraction and resolution
  - For an optical system, the aperture could be the observer's eye or the objective lens of a telescope.
  - When there are two sources of light, then two diffraction pattern will occur.
  - The Rayleigh criterion is used to determine the resolution of images.
    - The criterion states that two sources are resolved if the principle maximum from one diffraction pattern is no closer than the first minimum of the other pattern.
  - The limit to resolution is when the principle maximum of the diffraction pattern from one source lies on the first minimum diffraction pattern from the second source.
    - Any closer and they would be unresolved.
    - Further away and they'll be resolved.



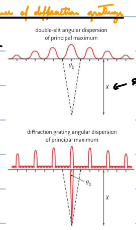
▲ Figure 1 Diffraction intensity patterns of two objects viewed through a circular aperture.

- Resolution equation
  - For a circular aperture the equation for the minimum is given by: $\theta = 1.22\frac{\lambda}{a}$.
    - In this case a is the diameter of the circular aperture (or diameter of the lens).
    - An example would be the eye which has a diameter of 3 mm, with light of $6 \cdot 10^{-7}$ m.
      - The angle of resolution is: $\theta = \frac{(1.22)(6 \cdot 10^{-7})}{3 \cdot 10^{-3}} \approx 2 \cdot 10^{-4}$ rad.
  - Another example would be the Hubble Space Telescope with a diameter of 2.4 m.
    - The minimum angle of resolution is $\theta = \frac{(1.22 \cdot 6 \cdot 10^{-7})}{2.4} = 3 \cdot 10^{-7}$ rad.
  - Worked example
    - $\lambda = 550 \cdot 10^{-9}$ m, $\theta = 2.5 \cdot 10^{-4}$ radian
    - The Rayleigh criterion states that an image can be resolved if the first minimum isn't any closer than the first maximum of the second source.
    - $2.5 \cdot 10^{-4} = \frac{1.22\lambda}{a}$

$a = \frac{1.22(550 \cdot 10^{-9})}{(2.5 \cdot 10^{-4})}$

$= 2.68 \cdot 10^{-3}$ m

- Resolvance of diffraction gratings
  -



▲ Figure 3 Angular dispersion of principal maximum for diffraction grating compared with a double-slit.

- As seen, the angular dispersion of the double slit will be larger than the angular dispersion of the diffraction grating.
    - A sharper principal maximum is one with less angular dispersion.
    - With a wider maxima there is more overlap of images from different sources and lower resolution.
- With wider maxima there is more overlap of images from different sources and lower resolution.
- This can be seen when a wider beam of light is incident on a diffraction grating, it will produce a sharper image and better resolution.
- The resolvance R for a diffraction grating is defined as: the ratio of the wavelength λ of the light to the smallest difference in wavelength that can be resolved by the grating $\Delta\lambda$
    - The resolvance is equal to Nm where N is the total number of slits illuminated by the incident beam and m is the order of the diffraction.
    - $R = \frac{\lambda}{\Delta\lambda} = Nm$
- Worked example
    - $\lambda_1 = 589 \cdot 10^{-9}$ m $\lambda_2 = 589.6 \cdot 10^{-9}$, $m = 2$, $0.1 \cdot 10^{-3}$ m

      $R = \frac{589 \cdot 10^{-9}}{0.6 \cdot 10^{-9}} = 2N$

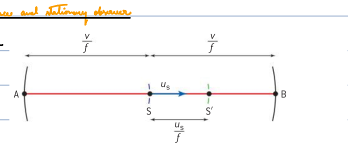
      $N = 491$ ← Beam of 0.1 mm^{-3}

      $N = 4910$ lines per mm

      $\approx \underline{5000}$

9.5 The Doppler Effect

- The Doppler effect with sound waves
- "s" means the source of the wave, while "o" is the observer.
    - F is the frequency of the source, and f' is the apparent frequency measured by the observer.
- Moving source and stationary observer
    -



▲ Figure 1 The Doppler effect for a moving source and stationary observer

- At the time $T = 0$ the source is at position A and it emits a wave that travels outwards in all directions, with a velocity v.
    - After $t = T$ (i.e. one period later) the wave will have moved one wavelength or a distance equivalent to $\frac{v}{f}$ to go from A to B.
    - The source is moving to the right with velocity u_s.
        - In time T the source will have arrived at point A', a distance of $u_s T$ or $\frac{u_s}{f}$.
- For the observer at point B, the wavelength (λ') is the distance $\underline{\underline{A'B = \frac{v}{f} - \frac{u_s}{f}}}$.
    - Which will equal: $\lambda' = \frac{v - u_s}{f}$
    - Therefore the frequency will appear to be higher to the observer as the object moves towards them.
    - The sound wave will travel at speed v so $f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - u_s}\right)$.
- On the other hand, the frequency will seem to be lower to the observer at point A as the source moves away.
    - $\lambda' = \frac{v + u_s}{f}$, and $f' = \frac{v}{\lambda'} = f\left(\frac{v}{v + u_s}\right)$.
- Combining the two equations will result in: $f' = f\left(\frac{v}{v \pm u_s}\right)$.
    - "−" for when the source is moving towards the observer, and "+" when its leaving.
- Moving observer and stationary source
    - In this scenario the source is still while the observer moves towards it, with a velocity u_o.
    - The source emits waves at a frequency f but the observer, moving towards the source, will experience a higher frequency.
        - The observer will observe that the waves are travelling with velocity $v + u_o$ and frequency f'.
    - The frequency measured by the observer is: $f' = f\left(\frac{v + u_o}{v}\right)$, when the observer is moving towards the source.
        - When the observer is moving away from the the source the frequency equation will be:
            - $f' = f\left(\frac{v - u_o}{v}\right)$.
    - Combining the equations will be: $f' = f\left(\frac{v \pm u_o}{v}\right)$.
- REMEMBER!!! u_o is the velocity of the observer, v is the speed of sound, f is the frequency of the sound, u_s is the velocity of the source.
- Worked example
    - $f = 2000$ Hz, $u = 15$ m s^{-1}, v (sound) $= 330$ m s^{-1}
    - The loudspeaker is constantly undergoing the doppler effect as it moves closer and further away, resulting in the frequency appearing to increase and decrease.
    - $f' = (2000)\left(\frac{330}{315}\right)$

      $= 2095$ Hz

      $\approx \underline{2100 \text{ Hz}}$
- The Doppler effect with light

- $\Delta f \approx f \frac{v}{c}$
    - c = speed of light, f = frequency of light, v = velocity of object
    - the equation is also equal to $\Delta\lambda \approx \lambda \frac{v}{c}$.
- <u>Radar</u>
    - the $\Delta f = f \frac{v}{c}$ equation changes to $\Delta f = 2f \frac{v}{c}$.
- <u>Worked example</u>
    - $v = 1.9\ \text{kms}^{-1}$, $\lambda = 587.5618\ \text{nm}$
        - $\Delta\lambda = \frac{\lambda v}{c}$

          $= \frac{(587.5618 \cdot 10^{-9})(1.9 \cdot 10^{3})}{3 \cdot 10^{8}}$

          $= \underline{3.72 \cdot 10^{-12}\ \text{m}}$
    - $\lambda = 150 \cdot 10^{-3}$, $\Delta f = 5 \cdot 10^{3}\ \text{Hz}$

      $c = f\lambda$

      $f = \frac{c}{\lambda}$

      $= \frac{3 \cdot 10^{8}}{150 \cdot 10^{-3}}$

      $= 2 \cdot 10^{9}\ \text{Hz}$

      $\Delta f = 2f \frac{v}{c}$

      $\underline{v = 375\ \text{ms}^{-1}}$